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ABSTRACT

The index of refraction of a free electron laser (FEL) pulse is derived directly, independently of the general FEL analysis, by considering the propagation of an electromagnetic wave along an electron beam that is penetrated by a static magnetic field.

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Recent work[1] on the optical guidance of a free electron laser (FEL) pulse utilized the index of refraction of the pulse, as calculated in the course of a theoretical analysis of FEL behavior.[2] However, if the FEL electron pulse is regarded as the propagation medium for an electromagnetic wave, the index of refraction is a basic characteristic of the medium. As such, it may be calculated directly, independently of the general FEL analysis.

To show this, consider the following one-dimensional problem: an electron beam, moving in the z-direction with a velocity close to c, is penetrated by a static magnetic field, periodic in z and oriented transversely to z; what index of refraction does this medium present to an electromagnetic wave propagating along z?

Suppose the wiggler field, B_w, is oriented along x and the propagating electric field E_s is oriented along y. Then, the fields can be derived from two vector potentials,[3] oriented along y, given by

$$A_w = B_w \frac{\sin \int k_w \, dz}{k_w} \quad ,$$

and

$$A_s = E_s \frac{\sin \psi_s}{k_s} \quad ,$$

where,

$$\psi_s = k_s z - w_s t + \varphi(z) \quad .$$

Since there is no variation transverse to z, the canonical momenta along x and y are constants of the motion, which, as in reference 2, we take to be zero.

The y-velocity induced in the electron beam can be obtained directly from the canonical momentum along y:

$$v_y = \frac{-e}{\gamma mc}(A_w + A_s) \quad ,$$

and the polarization current along y, is (η = index of refraction)

$$nev_y = \frac{\dot{P}}{4\pi} = \frac{(\eta^2-1)}{y}\frac{\dot{E}_y}{4\pi} = \frac{(\eta^2-1)}{c}(-\ddot{A}_s) \quad ;$$

or, writing ω_p for the plasma frequency,

$$(\eta^2-1)A_s = -\frac{\omega_p^2}{\omega_s^2}(A_w + A_s) \quad ,$$

which can be put in the form ($A_w >> A_s$),

$$(\eta^2-1)\frac{E_s}{k_s}(1-e^{-2i\psi_s}) = \frac{\omega_p^2}{\omega_s^2}\frac{e^{-i\psi}}{\gamma}\frac{B_w}{k_w}(1-e^{2i\int k_w dz}) \quad ,$$

with

$$\psi = \int k_w\, dz + \psi_s \quad .$$

Prosnitz et. al assume that ψ varies slowly, while ψ_s and $\int k_w\, dz - \psi_s$ vary rapidly. The rapidly varying terms may be averaged out, leaving

$$\eta^2-1 = \frac{\omega_p^2}{\omega_s^2}\frac{e^{-i\psi}}{\gamma}\frac{B_w k_s}{E_s k_w} \quad .$$

If, as in the FEL, $\eta \sim 1$, η^2-1 can be replaced here by $2(\eta-1)$. Since η can be complex, taking real and imaginary parts reproduces the results in reference 1, namely,

$$(\eta - 1) = \frac{\omega_p^2}{2\omega_s^2} \left\langle \frac{e^{-i\psi}}{\gamma} \right\rangle \frac{B_w k_s}{E_s k_w} \quad ,$$

where the brackets <...> represent an average over all electrons in the pulse.

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References

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2. (a) N. M. Kroll, P. L. Morton, and M. W. Rosenbluth, IEEE J. Quantum Electron. 17, 1436 (1981); (b) D. Prosnity, A. Szöke, and V. K. Neil, Phys. Rev. A24, 1436 (1981).

3. These fields are as in reference 2b, except that we use Gaussian units.

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